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August 1994The determination of $|V_{cb}|$ from semileptonic inclusive decay ratesUlrich Nierste^{a*}^aPhysik-Department T31, Technische Universität München, 85747 Garching, Germany

We report on the determination of $|V_{cb}|$ from the comparison of the semileptonic inclusive decay rates of B- and D-mesons using the heavy quark symmetry of QCD. While the renormalization scale ambiguity does not allow for a reliable estimate of the quark masses, it almost cancels in the prediction for $|V_{cb}|$, which reads

$$|V_{cb}|(\tau_B/1.49 \text{ ps})^{1/2} = 0.036 \pm 0.005,$$

where the error stems from the uncertainty in the kinetic energy of the heavy quark inside the meson, in the experimental branching ratios, in QCD input parameters, and from scale uncertainties.

1. Introduction

The following talk is based on research done in collaboration with Patricia Ball [1].

In the recent years the study of HQET, the effective theory of QCD expanded in inverse powers of the heavy quark mass m_Q , has considerably enlarged our understanding of low-energy QCD. Bigi, Uraltsev and Vainshtein [2] have started to apply HQET to inclusive processes and have been succeeded by many authors. Two important statements about inclusive decays could be obtained: first, in leading order they are essentially free quark decays, and second, the leading corrections to the free quark decay are of order $1/m_Q^2$. These results stimulated new determinations of the quark masses m_c and m_b and the CKM matrix element $|V_{cb}|$ from the experimental measurements of the semileptonic branching ratios $B(D \rightarrow X e \nu)$ and $B(B \rightarrow X_c e \nu)$ [3–5]. Now the determination of $|V_{cb}|$ from *exclusive* decay rates requires the analysis of the few experimental data points near the endpoint of the lepton spectrum and moreover needs some model-dependent input on a formfactor. The *inclusive* decay rates to order $1/m_Q^2$, however, only involve two real parameters λ_1 and λ_2 , of which the latter

is well known from the $B^* - B$ mass splitting. On the other hand they are proportional to the fifth powers of the poorly known quark masses, which are moreover renormalization-scheme dependent quantities. These points will be discussed in the following section, where the analytic expression for the inclusive decay rate will be introduced. Section 3 will in detail describe the phenomenological analysis.

2. The inclusive decay rate

When physical observables are calculated with the help of HQET, at first some QCD Green function is matched to its counterpart in HQET. In the case of the semileptonic inclusive decay rate this is the self energy Σ of the decaying heavy quark with a lepton pair and a quark in the intermediate state:

$$\begin{aligned} \Sigma_{\text{QCD}}(m_Q, \mu_Q) = & \\ & C_1(m_Q, \mu_Q) \left[\Sigma_{\text{HQET}}^{(0)} + \frac{1}{2m_{Q,\text{pole}}^2} \Sigma_{\text{HQET}}^{(1)} \right] \\ & + C_2(m_Q, \mu_Q) \frac{1}{2m_{Q,\text{pole}}^2} \Sigma_{\text{HQET}}^{(2)} + O\left(\frac{1}{m_Q^3}\right), \quad (1) \end{aligned}$$

whose imaginary part is related to the desired rate via the optical theorem. In (1) C_1 and C_2 are Wilson coefficients and the HQET matrix elements

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read in terms of the heavy quark field h :

$$\begin{aligned}\Sigma_{\text{HQET}}^{(0)} &= \frac{1}{2M_Q} \langle \mathcal{M} | \bar{h} h | \mathcal{M} \rangle = 1, \\ \Sigma_{\text{HQET}}^{(1)} &= \frac{1}{2M_Q} \langle \mathcal{M} | \bar{h} (iD)^2 h | \mathcal{M} \rangle = \lambda_1, \\ \Sigma_{\text{HQET}}^{(2)} &= \frac{1}{6M_Q} \langle \mathcal{M} | \bar{h} \frac{g}{2} \sigma_{\mu\nu} F^{\mu\nu} h | \mathcal{M} \rangle = \lambda_2(\mu_Q),\end{aligned}$$

where \mathcal{M} is the heavy meson with mass M_Q . Heavy quark symmetry dictates that the λ_i 's are the same for $\mathcal{M} = B$ and $\mathcal{M} = D$. The matching scale μ_Q in (1) must be of order m_Q , where both perturbative QCD and HQET are valid. As indicated in (1) the expansion parameter in the HQET matrix elements is the QCD pole mass. The Wilson coefficients $C_1, C_2 \propto m_Q^5$, however, contain the mass parameter of the renormalization scheme chosen to calculate the QCD Green function on the left hand side of (1). In view of the fact that the Wilson coefficients contain the short distance physics from scales larger than μ_Q , while the interaction from lower scales is contained in the matrix elements, we have used a short distance mass $m_Q^{\overline{\text{MS}}}$ evaluated at the matching scale μ_Q in the C_i 's. Clearly, the fifth power of m_Q causes a sizeable dependence of the decay rate on the renormalization scheme used to define m_Q . Recently Beneke and Braun and Bigi et al. [6] have found that the perturbation series defining the pole mass m_{pole} suffers from an extra IR-renormalon imposing an ambiguity of order Λ_{QCD} onto m_{pole} . When the inclusive decay rate is expressed in terms of m_{pole} , the perturbation series multiplying m_{pole}^5 exhibits the same renormalon ambiguity, which, however, vanishes, when the rate is expressed in terms of some short distance mass [7].

Taking the absorptive part of (1) results in

$$\begin{aligned}\Gamma(\mathcal{M} \rightarrow X \ell \nu) &= \frac{G_F^2 (m_Q^R)^5}{192\pi^3} |V_{\text{CKM}}|^2 \cdot \\ &\left[\left\{ 1 - \frac{2}{3} \frac{\alpha_s^R(\mu_Q)}{\pi} g^R(x^R) + \frac{\lambda_1}{2m_Q^2} \right\} f_1(x^R) \right. \\ &\quad \left. - \frac{9\lambda_2}{2m_Q^2} f_2(x^R) + \mathcal{O}\left(\frac{1}{m_Q^3}, (\alpha_s^R)^2, \frac{\alpha_s^R}{m_Q}\right) \right] (2)\end{aligned}$$

Here R marks the renormalization scheme de-

pendent quantities. In the following section we exploit (2) for $(\mathcal{M}, m_Q^R, V_{\text{CKM}}, x^R) = (B, m_b^{\overline{\text{MS}}}(\mu_b), V_{cb}, m_c^{\overline{\text{MS}}}(\mu_b)/m_b^{\overline{\text{MS}}}(\mu_b))$ and $= (D, m_c^{\overline{\text{MS}}}(\mu_c), V_{cs}, m_s^{\overline{\text{MS}}}(\mu_c)/m_c^{\overline{\text{MS}}}(\mu_c))$. The matching scales $\mu_b \approx m_b$ and $\mu_c \approx m_c$ will be varied to judge the renormalization scale dependence. The analytic expressions for $g(x)$ and the phase space factors $f_i(x)$ can be found in [1].

3. The determination of $|V_{cb}|$

The input of our phenomenological analysis is similar to [3–5]. It consists of four steps:

Step 1: Extract $m_c^{\overline{\text{MS}}}(m_c)$ from the experimental result for $\Gamma(D \rightarrow X e \bar{\nu})$ as given in (2) as a function of $\mu_c, \lambda_1, m_s(1\text{GeV})$ and $\Lambda_{\overline{\text{MS}}}$.

Step 2: Get $m_b^{\overline{\text{MS}}}(m_b)$ from $m_c^{\overline{\text{MS}}}(m_c)$ via the heavy quark symmetry, which relates pole masses:

$$\begin{aligned}m_b^{\overline{\text{MS}}}(m_b) &= m_c^{\overline{\text{MS}}}(m_c) + \Sigma_b^{(1)} - \Sigma_c^{(1)} + m_B \\ &\quad - m_D + \frac{\lambda_1 + 3\lambda_2}{2m_b} - \frac{\lambda_1 + 3\lambda_2}{2m_c},\end{aligned}\quad (3)$$

where $\Sigma_q^{(1)}$ is the one-loop QCD quark self energy.

Step 3: Insert $m_b^{\overline{\text{MS}}}(m_b)$ into (2), but now for $\Gamma(B \rightarrow X_c e \bar{\nu})$, to find $|V_{cb}|$.

Step 4: Vary the two matching scales μ_c and μ_b to estimate the renormalization scale dependence and also the physical parameters $\lambda_1, m_s^{\overline{\text{MS}}}(1\text{GeV}) \dots$ to judge the total error of the theoretical prediction.

The one-loop expression for the decay rate (2) exhibits a large scale dependence, especially for the D-decay. This fact obscures the determination of the quark masses as displayed in fig. 1., for which we find

$$\begin{aligned}m_c^{\overline{\text{MS}}}(m_c) &= (1.35 \pm 0.20) \text{ GeV}, \\ m_b^{\overline{\text{MS}}}(m_b) &= (4.6 \pm 0.3) \text{ GeV}.\end{aligned}\quad (4)$$

This shows that one has to calculate higher orders in (2) in order to reduce the scale dependence, if one wants to extract quark masses from the inclusive decay rates. In the determination of $|V_{cb}|$ the scale dependence, however, reduces drastically as displayed in fig. 2. Here we have taken $\mu_c/m_c = \mu_b/m_b$, because by varying the scale we

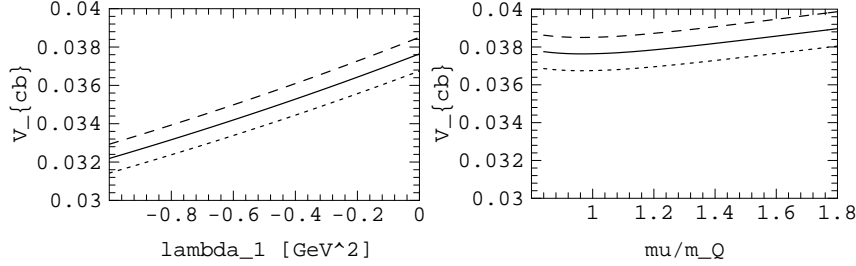


Figure 2. Left: $|V_{cb}|(\tau_B/1.49 \text{ ps})^{1/2}$ vs. λ_1 for $\mu_Q = m_Q$. Right: $|V_{cb}|(\tau_B/1.49 \text{ ps})^{1/2}$ vs. μ_Q/m_Q for $\lambda_1 = 0$. Solid line: $B(D \rightarrow X e \nu) = 0.172$, $m_s(1 \text{ GeV}) = 0.2 \text{ GeV}$ and $\Lambda_{\overline{\text{MS}}}^{(4)} = 300 \text{ MeV}$ and $B(B \rightarrow X_c e \nu) = 0.107$. The dashed lines correspond to the experimental error in $B(B \rightarrow X_c e \nu)$: $B = 0.102$ (short), $B = 0.112$ (long).

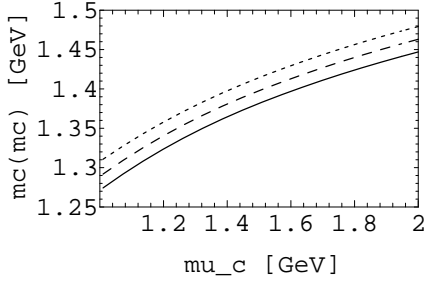


Figure 1. $m_c(m_c)$ vs. the renormalization scale μ_c for the branching ratio $B(D \rightarrow X e \nu) = 0.172$, $m_s(1 \text{ GeV}) = 0.2 \text{ GeV}$, $\Lambda_{\overline{\text{MS}}}^{(4)} = 300 \text{ MeV}$. Solid, long-, short-dashed line: $\lambda_1 = 0 \text{ GeV}^2, -0.35, -0.7 \text{ GeV}^2$.

want to estimate the importance of the yet uncalculated higher order terms in the perturbation series in (2), as the scale dependence vanishes order by order in perturbation theory. These uncalculated terms are of course the same function of μ_Q/m_Q for $\Gamma(B \rightarrow X_c e \bar{\nu})$ and for $\Gamma(D \rightarrow X e \bar{\nu})$ apart from the small effect that one has five active flavours in the former rate and four in the latter. Suppose the $O(\alpha_s^2)$ -corrections enhance Γ in (2): Then both the error and the central value for m_c and m_b in (4) obtained in step 1 and 2 will be lower, but in step 3 this lower value for m_b^5 will multiply a larger radiative correction to Γ in (2), thereby stabilizing the prediction for $|V_{cb}|$.

Finally the largest uncertainty in $|V_{cb}|$ originates from λ_1 as shown in fig. 2. We have varied

λ_1 between -0.7 GeV^2 [8] and 0 [9] and find:

$$|V_{cb}| \left(\frac{\tau_B}{1.49 \text{ ps}} \right)^{1/2} = 0.036 \pm 0.005.$$

This has to be compared with

$$|V_{cb}| \left(\frac{\tau_B}{1.49 \text{ ps}} \right)^{1/2} = (0.046 \pm 0.008) \quad [3],$$

$$|V_{cb}| \left(\frac{\tau_B}{1.49 \text{ ps}} \right)^{1/2} \approx 0.042 \quad [4].$$

We remark that in [3] the renormalization scale has been varied down to $\mu_c \approx 0.5 \text{ GeV}$, which is too low to trust into perturbative QCD. Recent exclusive measurements (CLEO) gave [10]:

$$|V_{cb}| = 0.0362 \pm 0.0053.$$

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Elsevier instructions for the preparation of a 2-column format camera-ready paper in \LaTeX

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